Analysis of Ferrocement Slabs Using Finite Element Method

Ihsan Qasim Mohamad

University of Basrah, College of Engineering, Department of Civil Engineering
Iraq, Basrah

Abstract

This paper is concerned with the application of finite element techniques to the nonlinear analysis of ferrocement slabs. Both material and geometric nonlinearities are considered in the analysis. Concrete compression is modelled by a plasticity model and smeared cracking approach is used for tensile cracking. Degenerated thick shell elements employing a layered discretization through the element are adopted. Analyzing of a ferrocement slab does validation of the proposed model.

.

تحليل السقوف الفيروسمنتية باستعمال طريقة العناصر المحددة

إحسان قاسم محمد
جامعة البصرة – كلية الهندسة – قسم الهندسة المدنية
العراق – البصرة

الخلاصة

يعنى هذا البحث بتطبيق تقنية العناصر المحددة في التحليل اللاخطي للسقوف الفيروسمنتية. تم الأخذ بنظر الاعتبار التغير اللاخطي للمادة والشكل. تم تمثيل انضغاط الخرسانة باستخدام نموذج لدن، أما بالنسبة للخرسانة في حالة الشد فقد استخدم مبدأ توزيع الشقوق لتمثيل تشقق الشد. كما استخدمت العناصر القشرية السميكة المكبوسة مع التقسيم خلال سمك العنصر. تم تمثيل سقف فيروسمنتي لاختبار قابلية البرنامج المطور وكفاءة النموذج المستخدم.

1. Introduction

The last years had shown significant steps in research in order to develop ferrocement as an active building material. Ferrocement is a form of reinforced concrete made of wire mesh and mortor, which has unique values of serviceability and strength. It can be constructed from easily available materials and it does not require a large number of skilled workers. It has many applications and uses in the area of low-cost houses, agriculture, and industry.

In order to ensure the serviceability and strength requirements of ferrocement slabs, it is necessary to accurately predict the overall deformational properties during the range of their elastic and plastic response accurately, as well as the ferrocement slabs strength at ultimate collapse. Although the need for experimental research to provide the basis for design equations continues, the development of powerful and reliable analytical techniques, such as finite element method, can reduce the time and cost of otherwise expensive experimental tests, and may better simulate the loading and support conditions of the actual structure. Accurate results of finite element analysis, however, require adequate modeling of the actual behavior of the materials including nonlinearity.

Ferrocement exhibits nonlinearity because of cracking, inelastic material behavior, stiffening and softening phenomena, complexity of bond between wire mesh and concrete and other factors.

In the last 25 years several approaches have been used for the analysis of ferrocement structures, but the first use of finite element technique to analyze such structures was done by Prakhya and Adidan [1], who analyzed ferrocement slabs using rectangular hetrosis elements.

Bin-Omer et al. [2] presented a computational model based on the Timoshenko beam finite element formulation using quadratic isoparametric elements with 3 degrees of freedom to analyze flanged ferrocement beams.

Boshra Aboul-Anen et al. [3] used ANSYS software with Eight-node solid isoparametric element to study the composite action between the ferrocement slabs and steel sheeting.

The objective of this paper is to develop a non-linear layered shell element model to simulate the behavior of ferrocement slabs.

The validity and calibration of the theoretical formulations and the program used is judged through comparison of analytical results with available experimental data.

2. Modeling of Material Properties

The nonlinear components considered in the present model are:

1. plastic flow of concrete with hardening according to a prescribed yield criterion in terms of stress invariants,
2. smeared concrete cracking with tension stiffening effects,
3. shear degradation,
4. crushing of concrete, and
5. elasto-plastic behaviour of steel reinforcement.

2.1 Concrete

An elastic-plastic material model is used for the compression behaviour of the concrete. This is the same as the one used by Owen and Figueiras [4]. The yield function is expressed as a function of two stress invariants I_1 and J_2 as:

$$f(I_1, J_2) = (\alpha I_1 + 3\beta J_2)^{1/2} = \sigma_o \qquad \ldots(1)$$

were σ_o is the equivalent effective stress taken as the uniaxial compressive strength. The material parameters α and β can be obtained by fitting biaxial test results of Kupfer et al. [5]. These constants are $\alpha = 0.355$ and $\beta = 1.355$ and the yield function expressed in terms of stress components is:

$$f(\sigma) = [0.355\sigma_o(\sigma_x + \sigma_y) + 1.355\{(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y) + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\}]^{1/2} = \sigma_o \quad ...(2)$$

The comparison of the above expression with the experimental results of Kupfer et al. [5] in biaxial stress space is shown in Figure 1.

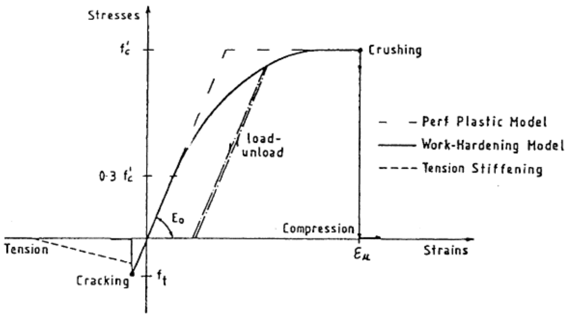

Figure 1 – Compressive Yield and Tensile Cracking Criterion for Concrete (Biaxial Stress State) [4].

In order to construct the stress-strain relationship in the plastic range, assuming normality of the plastic deformation rate vector to the yield surface in the form:

$$d\varepsilon_p = d\lambda \frac{\partial f(\sigma)}{\partial \sigma} \quad \ldots(3)$$

The elasto-plastic constitutive matrix is:

$$[D]_{ep} = [D] - \frac{[D]\{a\}\{a\}^T[D]}{H' + \{a\}^T[D]\{a\}} \quad \ldots(4)$$

where $[D]$ is a (5×5) elastic matrix, $\{a\}$ is a (5×1) flow vector with

$$\{a\}^T = \frac{\partial f(\sigma)}{\partial \sigma} \quad \ldots(5)$$

and H' is the slope of the effective stress-plastic strain curve.

The concepts of effective stress and effective plastic strain are used to extrapolate from the results of a uniaxial test to a multiaxial situation.

For defining the uniaxial stress-strain relation, the conventional " Madrid parabola" is used [4],

$$\sigma_o = E_o\,\varepsilon - \frac{E_o}{2\,\varepsilon_o}\,\varepsilon^2 \quad \ldots(6)$$

where E_o is the initial elasticity modulus, ε_o is the strain at peak stress, and ε is the total strain. Equation (6) may be expressed in terms of plastic strain " ε_p " in the form :

$$\sigma_o = -E_o\varepsilon_p + \varepsilon_o(2\,\varepsilon_o\,\varepsilon_p)^{0.5} \; for \quad 0.3 f_c' \le \sigma_o \le f_c' \quad \ldots(7)$$

using Equation (7), H' can now written as:

$$H' = \frac{d\sigma_o}{d\varepsilon_p} = E_o\{(\frac{\varepsilon_o}{2\,\varepsilon_p})^{0.5} - 1\} \quad (8)$$

The crushing criterion used is :

$$[\alpha I_1' + \beta(3J_2')]^{0.5} = \varepsilon_u \quad \ldots(9)$$

where I_1' and J_2' are strain invariants and ε_u is ultimate strain for uniaxial case. Using the material constants α and β determined from Kupfer's results, the crushing condition is expressed in terms of strain components as :

$$0.355\,\varepsilon_u(\varepsilon_x + \varepsilon_y) + 1.355[(\varepsilon_x^2 + \varepsilon_y^2 - \varepsilon_x\varepsilon_y) + 0.75(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)] = \varepsilon_u^2 \quad ...(10)$$

In tension, concrete is assumed to behave as a linear elastic material and the smeared crack approach is used with fixed orthogonal model. The tension stiffening effect of concrete is included in the analysis by allowing concrete stress normal to the crack to gradually fall down over a specified range as shown in Figure 2.

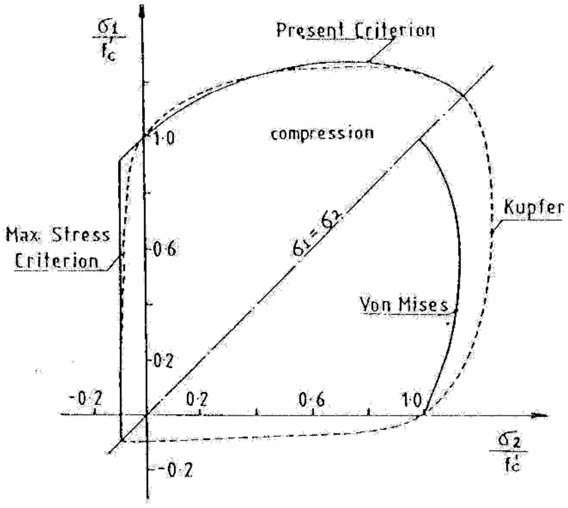

Figure 2 – Tension Stiffening Behaviour for Concrete [4].

Other components contributing to nonlinear behaviour of reinforced concrete were taken as described in detail in Reference [4].

2.2. Wire Mesh

In the present study, the wire mesh smeared into equivalent steel layers with uniaxial properties. Elasto-plastic behaviour with linear strain hardening and elastic unloading-reloading in the plastic range were assumed.

3. Finite Element Formulation

The eight-noded degenerated thick shell element introduced by Ahmed et al. [6] with five degrees of freedom per node is used in the analysis (Figure 3).

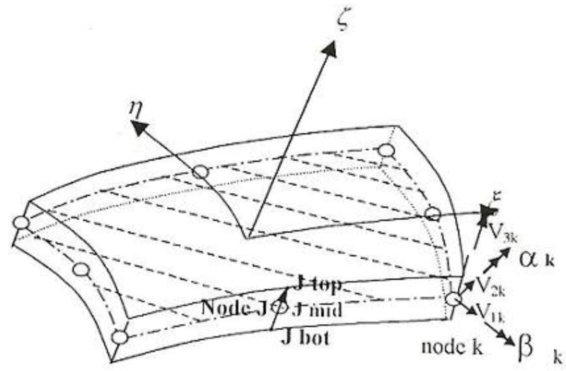

Figure 3 – Nodal Coordinate System of Degenerated Shell Element.

The process of degeneration is well known and in this context it will not be described in detail.

A total Lagrangian approach [4] is used to take into account the geometric nonlinear effects. In this approach, the initial geometry is taken as the reference configuration. The geometric nonlinearity is due to the second-order terms in the strain expressions with Von-Karman assumptions. A combined incremental-iterative technique is used to solve the nonlinear problem. Modified Newton-Raphson method is used where the stiffness matrix is updated at the second iteration of each load increment.

4. Example - Ferrocement Slab.

Analysis of ferrocement slab is presented in this section. This slab is the first of tow slabs tested by Boshra Aboul-Anen, et al. [3], to study the composite action between the ferrocement slabs and steel sheeting. Mortor and steel mesh properties are shown in Table 1.

Table 1. Material Properties of the Slab.

The slab dimensions were 0.8 x 0.8 m and 0.025 m thickness. The slab contains two wire meshes as shown in Figure 4.

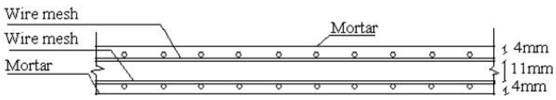

Figure 4 – Longitudinal Section in the Ferrocement Slab.

All four edges of the slab were simply supported over the span as shown in Figure 5.

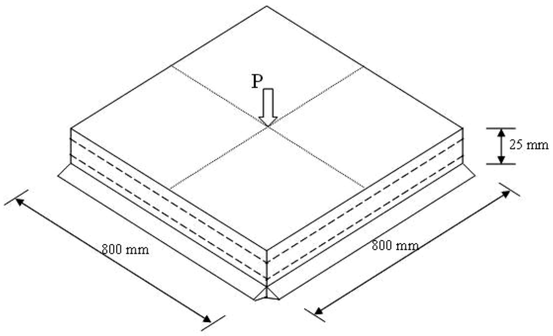

Figure 5 – Geometry and details for the slab.

Due to symmetry, only one quarter of the slab was analyzed. A mesh comprising 16 eight-nodded Serendipity element with a total of 65 nodes were used to model the slab. The finite element mesh is shown in Figure 6.

Mortor	Young's modulus E_{c28}, MPa	30000
	Compressive strength f'_{c28}, MPa	33.3
	Tensile strength f'_{t28}, MPa	2.00
	Poisson's ratio, υ	0.15
	Crushing Strain, ε_u	0.0035
Steel Mesh	Diameter, mm	1.42
	Grid size, mm	15x15
	Young's modulus, MPa	130000
	Yield stress, MPa	400
Tension Stiffening	α_m	0.6
	ε_m	0.002

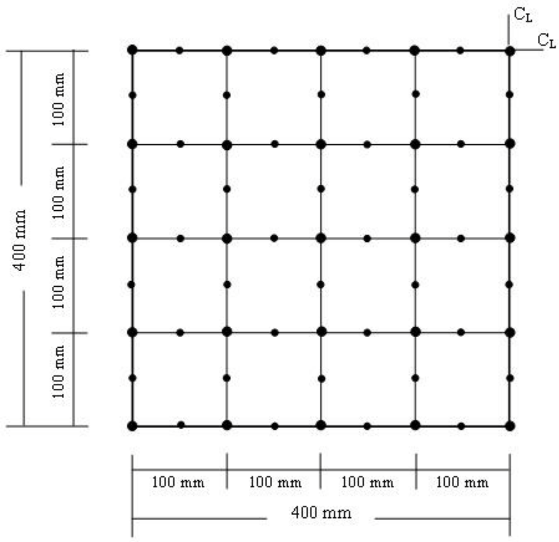

Figure 6 – Finite Element Mesh for the Slab.

Four mortor layers and tow wire mesh layers were used in the analysis.

5. Results:

The deflection versus applied load for the slab is shown in Figure [7].

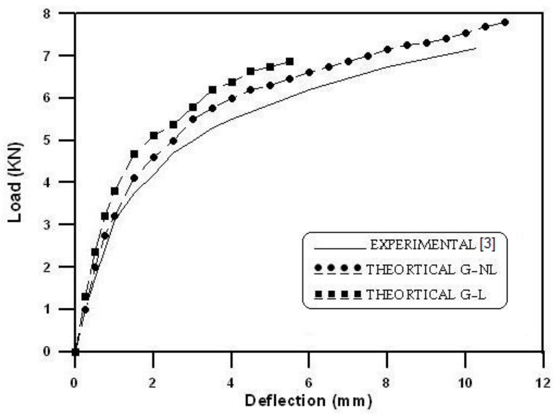

Figure 7 – Load Verses Deflection for the Slab at mid-span (Experimental and Theoretical), Considering Geometric Nonlinear Concept.

The analytical results are compared with the experimental data. Good agreement was obtained between the results especially when geometrical nonlinearity (G-NL) is taken into account. The geometric linearity (G-L) results given smaller ultimate load and smaller maximum deflection than the experimental results.

The ultimate load predicted by the theoretical model (G-NL) was higher than the experimental one with only 7.94 %. The comparison between theoretical and experimental load-central deflection relationship curve of the slab indicates a great matching between the two results up to 3.0 kN. After 3.0 kN, the experimental and theoretical results continue to a little mismatch until failure load. The numerical crack pattern of the ferrocement slab at failure is shown in Figure [8].

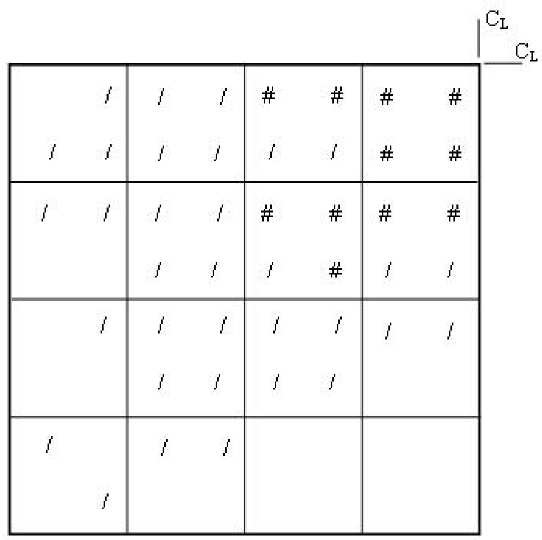

Figure 8 – The Numerical Crack Pattern for Bottom Layer of One Quarter of the Slab at Failure. (/ single crack, # doable crack)

6. Conclusions:

A procedure to analyze ferrocement slabs has been presented in this work. The validity of the proposed analytical model is assessed by comparing the numerical results with the available experimental data. The computational model is adequate for prediction the nonlinear behavior of ferrocement slabs under flexure up to failure. Geometric nonlinear concept plays an important role in predicting the analytical results and gives more accurate results.

References

1. K. V. G. Prakhya and S. R. Adidam, " Finite Element Analysis of Ferrocement Plates ", J. Ferrocement, No.17, 1978, pp. 313-320.
2. A. R. Bin-OMAR, H.H. Abdel-Rahman, G. J. Al-Sulaimani, " Nonlinear Finite Element Analysis of 'Flanged' Ferrocement Beams ", Computers & Structures, Vol. 31, No. 4, 1989, pp. 581-590.
3. Boshra Aboul-Anen, Ahmed El-Shafey, and Mostafa El-Shami, " Experimental and Analytical Model of Ferrocement Slabs " , International Journal of Recent Trends in Engineering, Vol. 1, No. 6, May 2009, pp. 25-29.
4. D. R. J. Owen and J. A. Figueiras," Ultimate Load Analysis of Reinforced Concrete Plates and Shells Including Geometric Nonlinear Effects", in Finite Element Software for Plates and Shells, by E. Hinton and D. R. J. Owen, Peneridge Press., Swansea 1984.
5. H. Kupfer, H. K. Hilsdorf and H. Rusch," Behaviour of Concrete Under Biaxial Stress", ACI J., Vol. 67, No. 8, Aug. 1969, pp. 656-666.
6. S. Ahmad, B. M. Irons and O. C. Zienkiewics," Analysis of Thick and Thin Shell Structures by Curved Finite Elements", Int. J. Num. Meth. Eng., Vol.2, No. 3, 1970, pp. 419-246.